Delta & Unit Step Functions

$$\frac{\partial u(t)}{\partial t} = \delta(t)$$

Convolution

$Y[n] \; = \; X_1[n] * X_2[n] = \Sigma_{k=-\infty}^{\infty} X_1[k]X_2[n-k]$

$X_1[n] * X_2[n] = X_2[n] * X_1[n]$

$\delta(t) * x(t) = x(t)$

$\Sigma_{k=-\infty}^{\infty} X[k]u[n-k]u[k] = \Sigma_{k=0}^{n} X[k]$

Fourier Series

$x(t) = \Sigma_{k=-\infty}^{\infty} a_k \, e^{jk\omega_0 t} = \Sigma_{k<T>} a_k \, e^{jk\omega_0 t}$

$a_n = T^{-1} \int_T x(t) \, e^{-jn\omega_0 t} \, dt$

Meaning K over the period.
(i.e. K=0 up to K=T… or any span of length T)

Inverse Fourier Transform

$x(t) = (2\pi)^{-1} \int_\infty X(j\omega) \, e^{j\omega_0 t} \, d\omega$

$x[n] = (2\pi)^{-1} \int_\infty X(e^{j\omega}) \, e^{j\omega_0 t} \, d\omega$

$X(j\omega) = \int_\infty x(t) \, e^{-j\omega t} \, dt$

e^x sinusoidal ↔ func. conversion

$\cos(\omega t) = .5 \, (e^{j\omega_0 t} + e^{-j\omega_0 t})$
$-\cos(\omega t) = .5 \, (-e^{j\omega_0 t} - e^{-j\omega_0 t})$
$\sin(\omega t) = .5 \, (e^{j\omega_0 t} - e^{-j\omega_0 t})$
$-\sin(\omega t) = .5 \, (e^{-j\omega_0 t} - e^{j\omega_0 t})$

Fourier Transform Convolution Property

If:
$x(t) \overset{F}{\leftrightarrow} X(j\omega)$
$y(t) \overset{F}{\leftrightarrow} Y(j\omega)$

Then:
$x(t) * y(t) \overset{F}{\leftrightarrow} X(j\omega)Y(j\omega)$

Integration By Parts

$\int f(x)g`(x) \, dx = f(x)g(x) - \int g(x)f'(x) \, dx$

Impulse Train Sampling

$p(t) = \Sigma_{k=-\infty}^{\infty} \delta(t - nT)$
T – sampling period
$\omega_s = 2\pi/T$ – sampling frequency

(sampled signal)
$x_p(t) = x(t)p(t)$